A STUDY IN THE CONCEPT OF PROBABILITY FILTERING

by

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THESIS

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ABSTRACT

The mathematical expectation formed from the first probability distribution lends itself well to implementation of a filtering device. This device, or probability filter, is discussed here conceptually and is computer simulated to obtain characteristics and performance of information in jamming and non-jamming environments.

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I. INTRODUCTION

An accepted and effective means of extrapolating signal from noise is by the use of correlation techniques. Cross correlation of signal with noise produces an improvement factor in favor of the signal but only upon payment of integration time and fairly complicated hardware. Correlation functions are directly related to the expectation or mean value of the functions being analyzed. The expectation of two input ensembles, X1 and X2, can be written as follows:

$$E\{X1,X2\} \equiv \int_{\infty}\int X1X2p(X1,X2;\tau)dX1dX2$$

where

$$X1 = f_1(t)$$

$$X2 = f_2(t+\tau)$$

and $p(X1,X2;\tau)$ = probability that ordinate pairs exist separated by τ seconds.

It is interesting to note that this expectation is equal to the cross correlation expression if the ensembles are ergodic and thus, the calculation is less cumbersome. If {X2} is noise, the expectation of signal with noise, E{X1,X2}, and thus the detrimental effects of noise, can be driven to zero by cross correlation.

However, this mathematical expectation can be formulated in a simpler way by using the first probability distribution:

$$E(X1) \equiv \int_{\infty} X1p(X1)dX1 \qquad (1)$$

where P(X1) = probability that X1 exists. In principle, the expectation of X1 can be made free of the influence of a disturbance signal, X2, by establishing a so called Region Of Expectation (R.O.E.) for the integrand of equation one. This region will be discussed in detail in

IIA. The region will "accept" a predefined expected range of X1 amplitudes but reject all others. To accomplish this we could construct a filtering device implementing equation one and rejecting X2 since it has random values larger in magnitude than the highest value in the R.O.E. Such a device may be termed a probability filter.

II. CHARACTERISTICS OF PROBABILITY FILTERING[1]

A. IMPLEMENTATION OF THE FILTER

The implementation of equation one and the study of the basic filter characteristics was accomplished by Figure 1(a). As shown, the filter is a two-block device composed of a probability weighting function block and an integration block. For the feedback loop to cause Θ_2 to track Θ_1 the probability filter must function independently of the input signal duration and must be sensitive to all incremental fluctuations in 'v'. Thus, the form of equation one is achieved and the filter forms the expectation as follows:

$$E(v) = \int_v vp(v)dv$$

The probability density function chosen for 'v' was the Gaussian density function with zero mean and unit variance. This function is shown sketched in Figure 2(a) and computer generated in Figure 3. This density function is expressed by:

$$p(v) = \frac{1}{\sqrt{2\pi}} e^{-v^2/2}$$

The integrand, v•p (v), is shown sketched in Figure 2(b) and computer generated in Figure 4. The Region Of Expectation is crosshatched in Figure 2(b) and has the following statistical properties:

1. Covers the area between a deviation (σ) of minus one and plus one.

2. Contains 78.6% of the total area of v•p(v).

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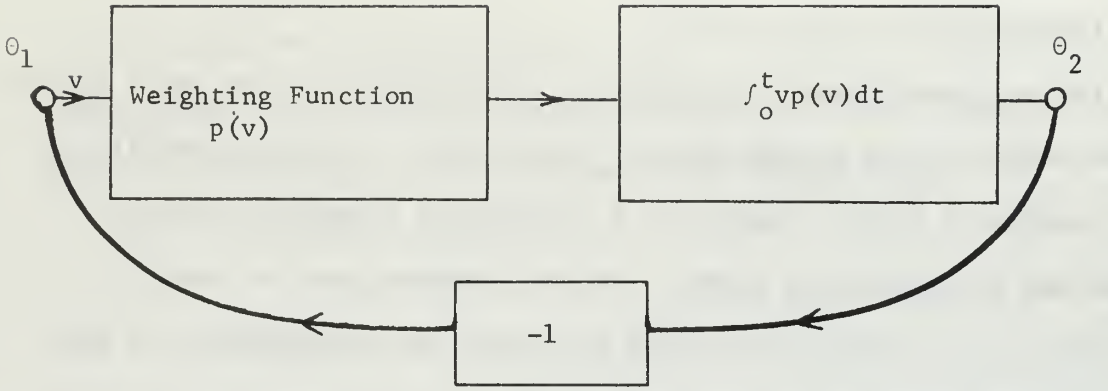
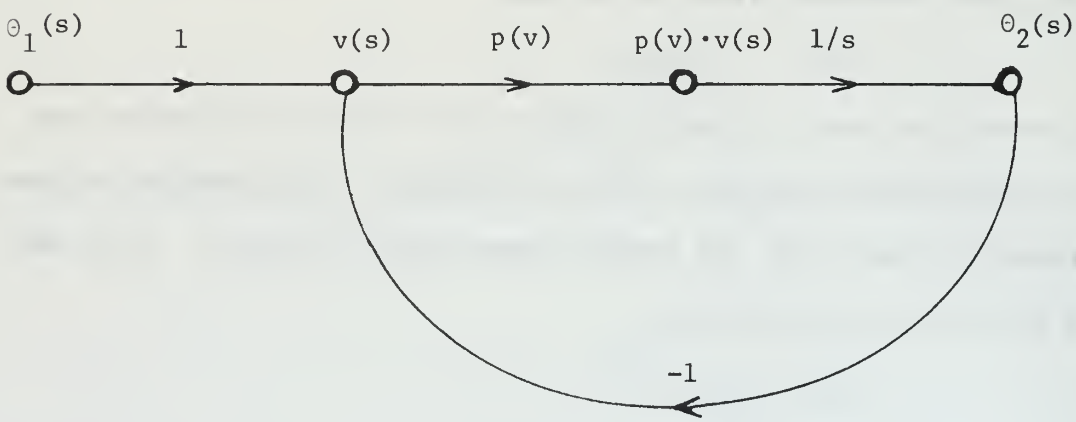
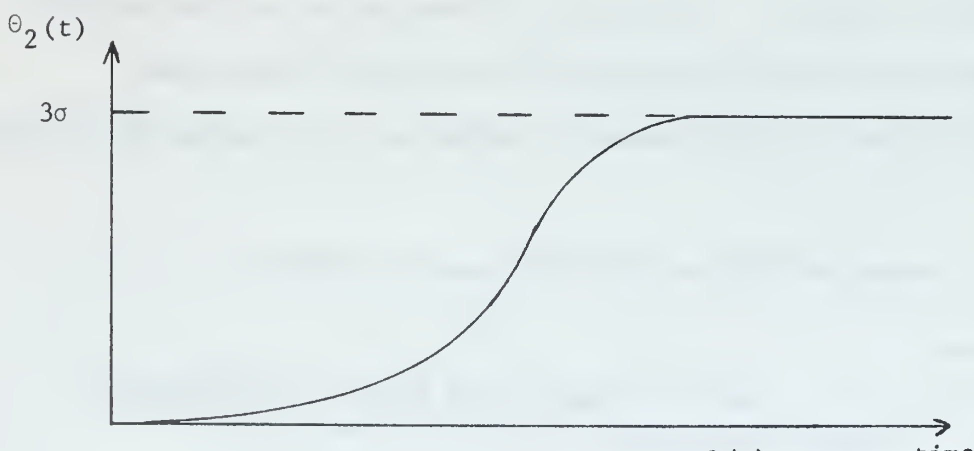
[1]Still, W. L., "Separate Signal from Noise with Probability Filters." Control Engineering, p. 147-151, March 1960.



(a) Probability filter in simple feedback loop.

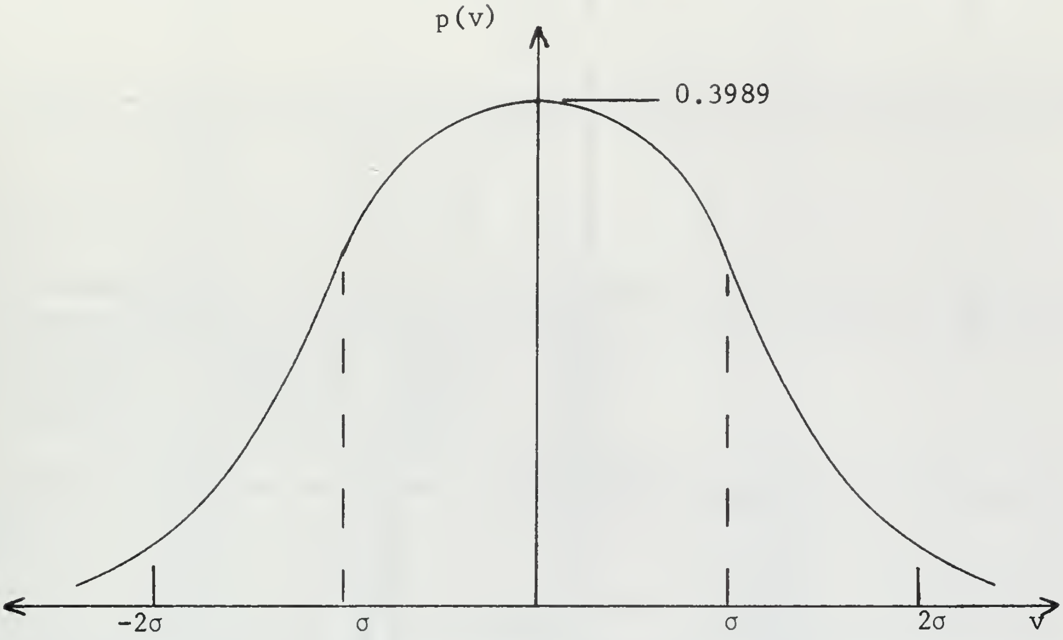


(b) Flow graph of 1(a)

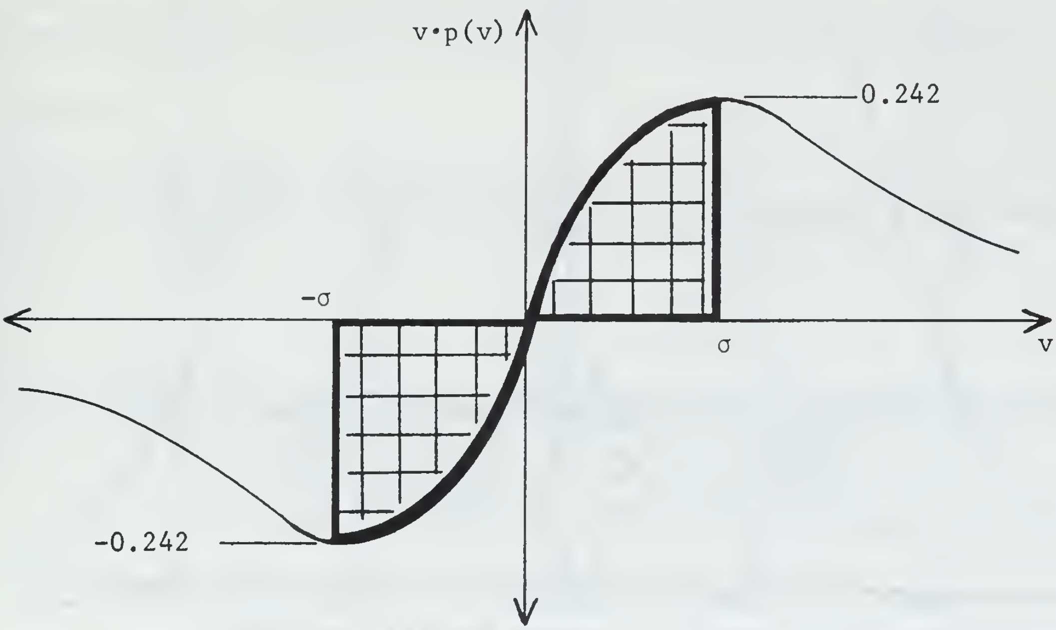


(c) Step response of 1(a)

Figure 1.



(a) Gaussian density function



(b) Expectation integrand vs filter input. Crosshatching indicates the Region Of Expectation (R.O.E.)

Figure 2.

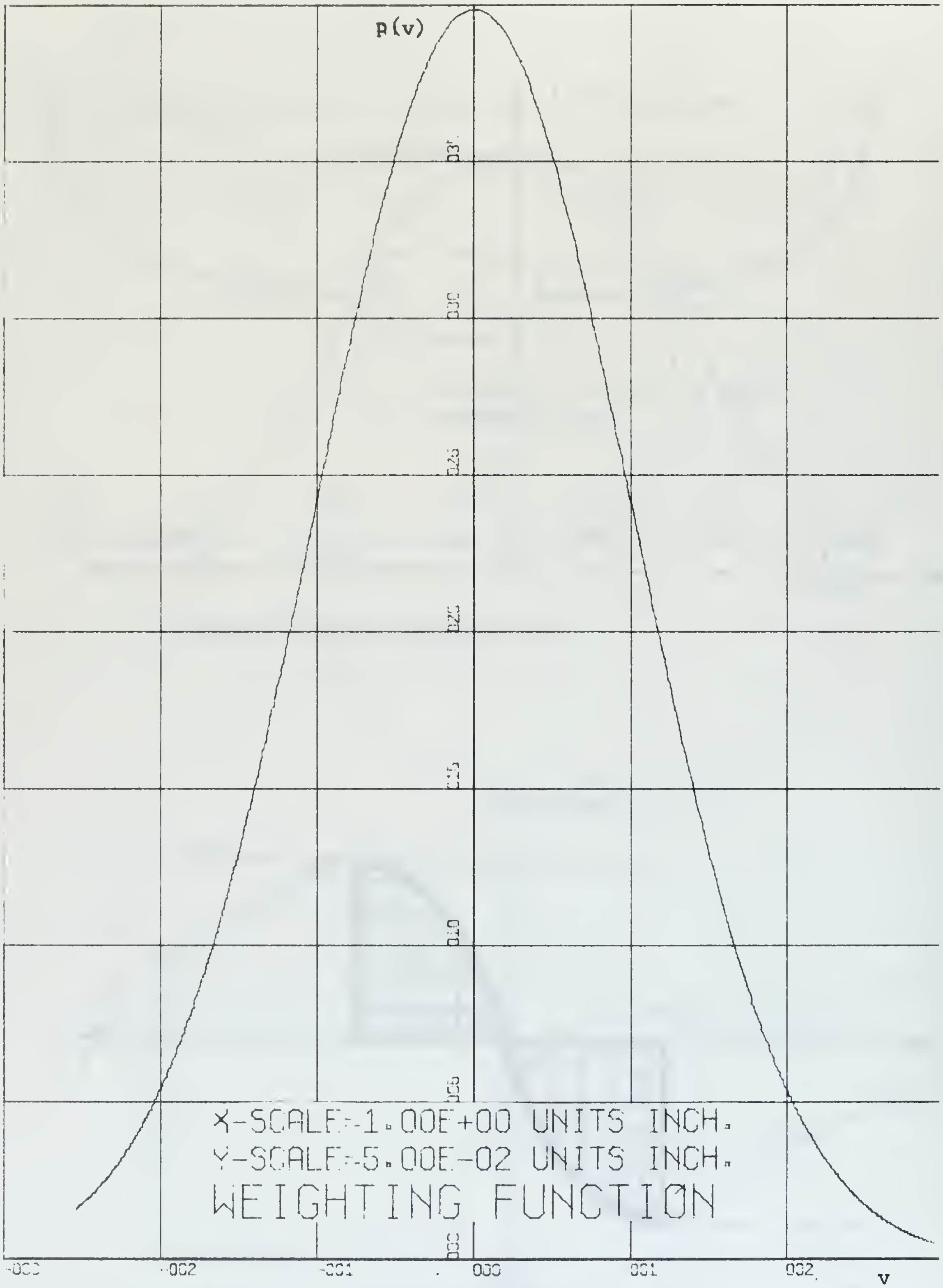


Figure 3. Computer generated Gaussian density function.

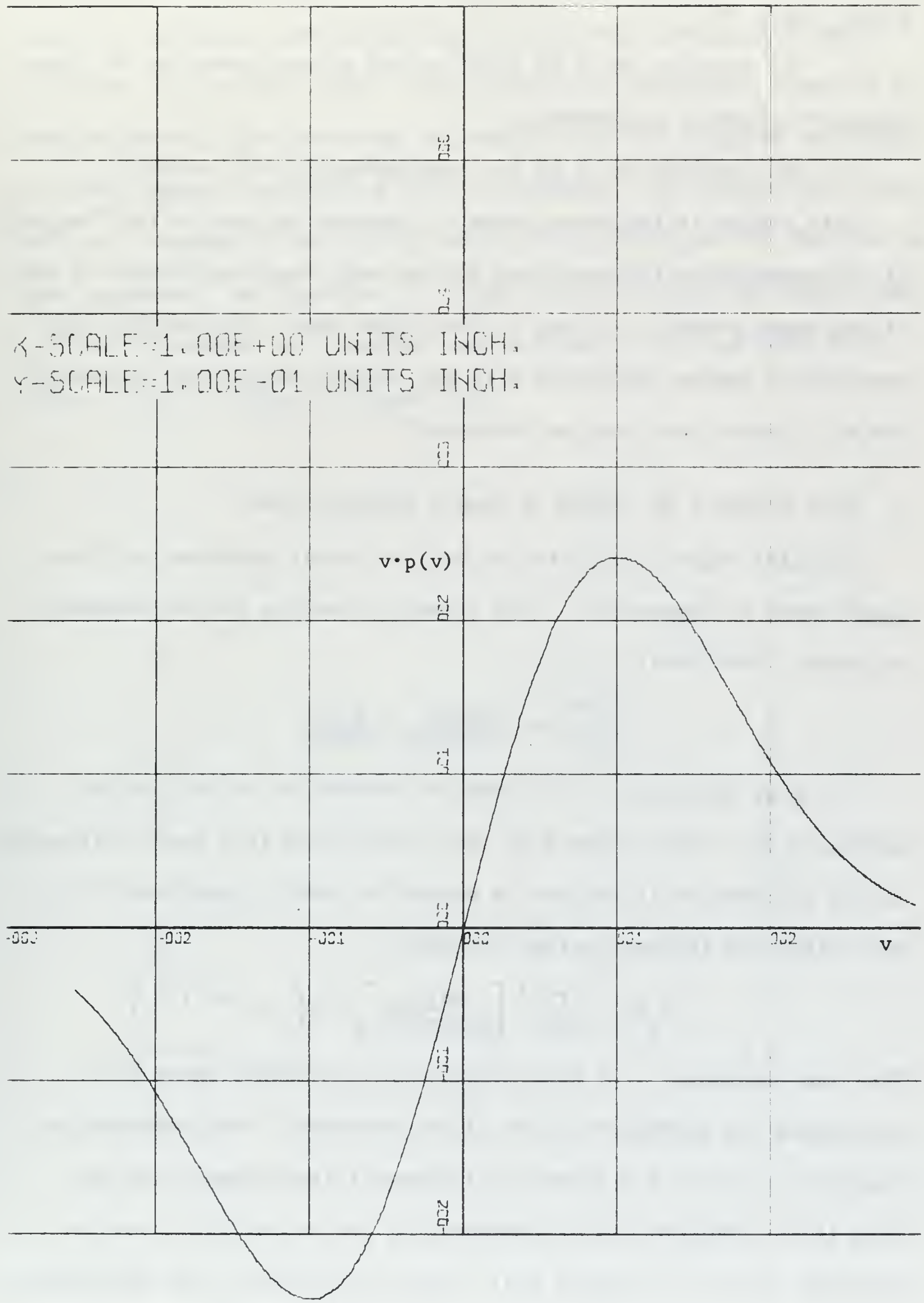


Figure 4. Computer generated plot of v·p(v) vs v

3. $v \cdot p(v)$ evaluated at $\sigma = \pm 1$ is equal to the rms value of the filter input 'v'.

4. The variance at ± 1 is equal to the average power in 'v' (providing 'v' lacks a d-c offset).

5. The variance at ± 1 is the expectation of 'v^2' or $E(v^2)$.

This region is important since it contains so much of the "weight" of the expectation integrand and defines many important values of the filter input signal. As long as the filter input stays within this expectation region the device will not attempt non-linear rejection and will regard the input as "expected".

B. STEP RESPONSE OF FILTER IN SIMPLE FEEDBACK LOOP

Changing Figure 1(a) into the Laplace domain produces the flow graph shown in Figure 1(b). The transfer function can be obtained by Mason's Gain Rule:

$$\frac{\Theta_2(s)}{\Theta_1(s)} = \frac{p(v)/s}{1+p(v)/s} = \frac{p(v)}{s+p(v)}$$

The step response of this feedback system can be analyzed by setting $\Theta_1(s) = X/s$, where X is the value of the step input referenced to one standard deviation and is assumed to have a magnitude of 3σ, and taking the inverse Laplace as shown:

$$\Theta_2(t) = \mathcal{L}^{-1}\left[\frac{xp(v)}{s(s+p(v))}\right] = X\left(1-e^{-p(v)\cdot t}\right)$$

The time constant, τ, of this response is $\tau = 1/p(v)$ where $p(v)$ represents the bandwidth of the filter response. This response is identical to that of a normal RC low-pass filter except that the τ term is not constant but is dependent on the probability density function, $p(v)$. The output will rise to the value of the input step, X, but the rise time constantly changes as the response transitions to its final value. An intuitive explanation of the response is as follows. Initially, the input error 'v' is large making p(v) quite small and τ relatively large. The system will, therefore, react in a sluggish manner until the error is reduced. As the response continues the error becomes smaller and the τ is reduced. The system then reacts more as a low-pass filter with a small RC time constant and the rate of rise increases. As the error 'v' falls into the Region Of Expectation $\Theta_2(t)$ will resemble the terminal step response of the low-pass RC filter. This is shown in Figure 1(c).

III. PROBABILITY FILTER APPLIED TO SECOND-ORDER SYSTEM

A. SECOND-ORDER SYSTEM MODEL

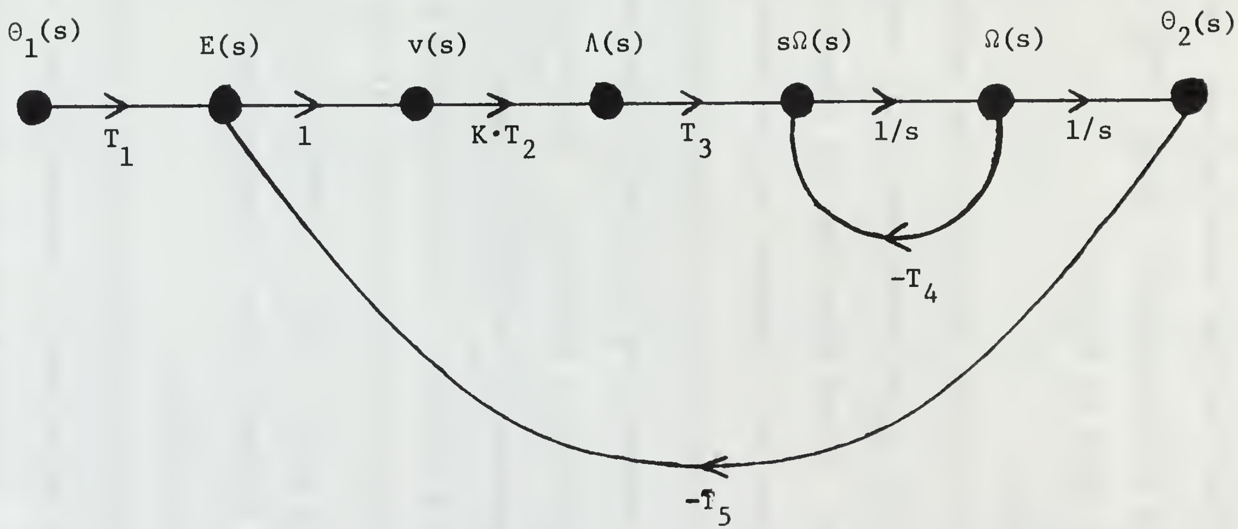
In order to investigate in depth the features of a probability filter a linear, second-order control system was modeled. The Laplace domain model is shown in Figure 5. The input is mechanically applied and converted to a voltage. This voltage, after comparison with the output position voltage, is amplified and converted to motor torque, $\Lambda(s)$. This torque accelerates a drive shaft, $s\Omega(s)$, through a gear train. The output shaft position settles to the input value when the feedback drives the system to a steady-state error, E(s), of zero. The transfer function of this system is as follows:

$$\frac{\Theta_2(s)}{\Theta_1(s)} = \frac{1560}{s^2+25s+1560}$$

The model was put into state variable form and computer programmed utilizing fourth-order Runge-Kutta integration. The system was tested with various step input values and the expected output responses were obtained. The rise time (time to reach the maximum value) was constant for all inputs and the overshoots varied in direct proportion to the value of the input step.

B. STEP RESPONSE WITH FILTER APPLIED

The probability filter was added between E(s) and V(s) in Figure 5, and the system was again computer programmed in state variable form. The six state variables utilized were the output position, $\Theta_2(t)$, and its first and second derivatives along with the error, E, the Filter Gaussian density function, P(E), and the expectation integrand, $E \cdot P(E)$.



$T_1 = T_5 \equiv$ Conversion factor, angle to voltage = 1.0.
$K \equiv$ Amplifier gain.
$T_2 \equiv$ Transfer factor, amplifier field current to motor torque.
$T_3 \equiv$ Transfer factor, motor torque to angular acceleration.
$T_4 = 25.0 \equiv$ Derivative feedback indicative of system friction.
$K \cdot T_2 \cdot T_3 = 1560.0$

Figure 5. Analysis model flow graph.

Various step inputs referenced to one standard deviation were again entered into the system. It was expected that the response for inputs within the R.O.E. would be linear and similar to those observed in IIIA. However, all step inputs less than one deviation in magnitude produced unstable outputs. Most outputs with forcing values greater than one deviation in magnitude were seen to be marginally stable. The instability with an input of 0.5σ is shown in Figure 6.

1. Correction of Instability

To correct the instability in the filter and also produce the desired linear response for filter inputs within the R.O.E. various feedback schemes were attempted. It was found that the source of instability was the integrator block and a feedback loop exclusively about this block was necessary to stabilize the filtered response over all input ranges. A standard feedback loop about both blocks was unsatisfactory for the implementation of stability. Said another way, the filter needed "leaky" integration for satisfactory operation. The stabilized filter together with a section of the model system is shown in Figure 7. For stability in filter integration point 'X' must be reduced in value by the feedback path. Without feedback this point has the value of $E \cdot P(E)$. With feedback on the integrator point 'X' becomes:

$$X = E \cdot p(E) - \alpha \int_0^t E \cdot p(E) dt$$

where $\alpha \int_0^t E \cdot p(E) dt$ forms the necessary reduction term for stability.

To achieve the response objectives, a feedback factor, α, of four was found to be necessary. This value was very critical and had no tolerance spread. Step responses for inputs within the R.O.E. were again observed and seen to be practically identical with those for the same inputs observed in IIIA. The overshoots in both cases correlated well (within 9%) up to 0.75σ and then increased to an 18%

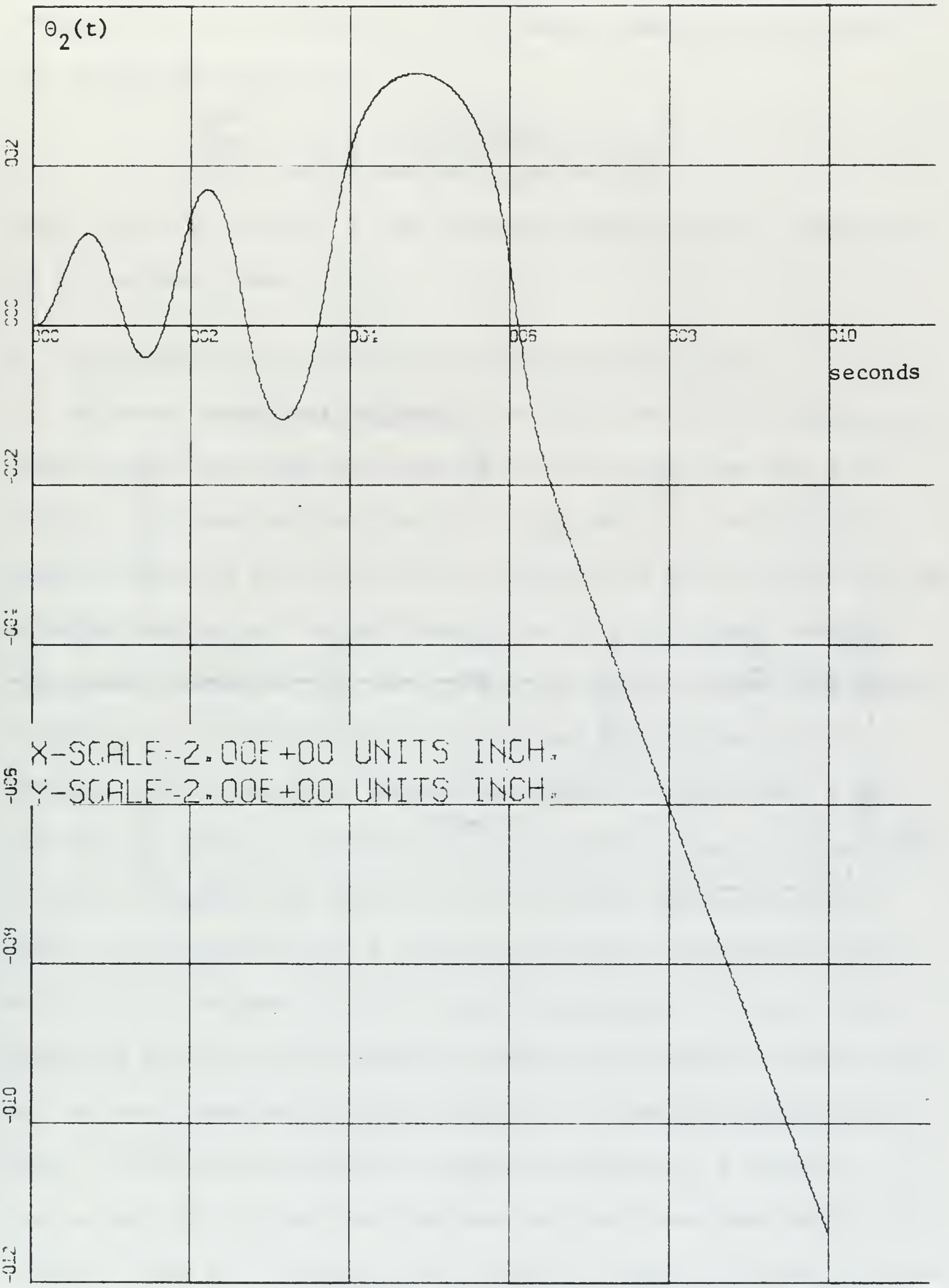


Figure 6. Probability filter instability for input of 0.5σ

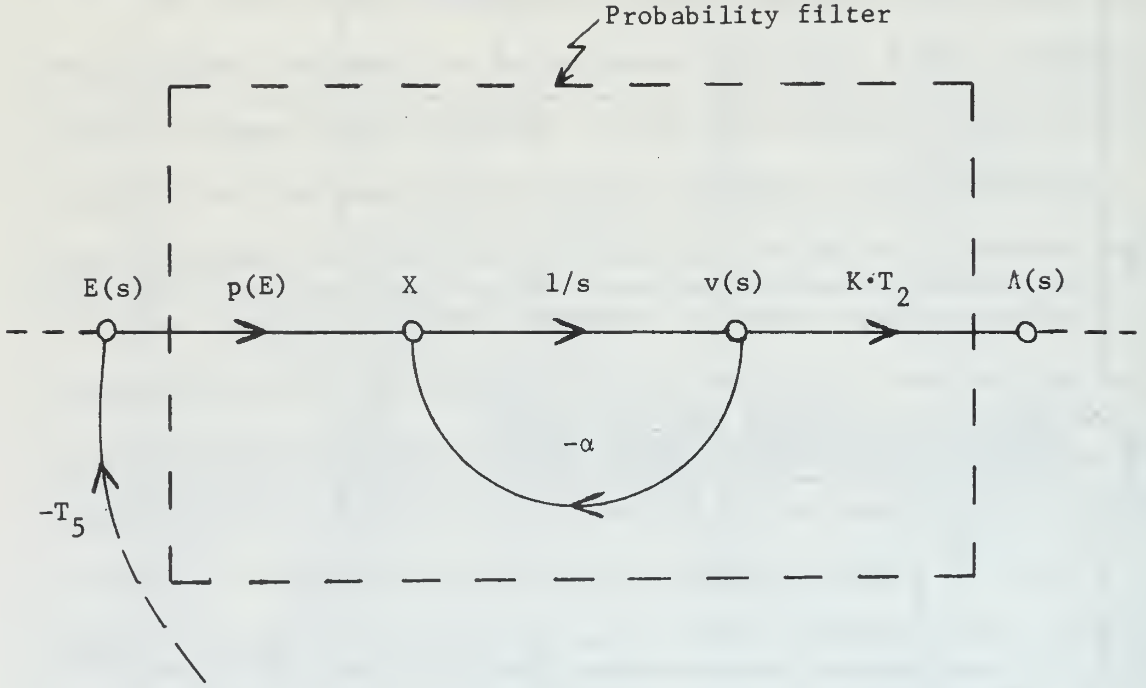


Figure 7. Stabilized filter configuration.

deviation near σ. The rise times remained nearly constant at 0.8 seconds ($\pm$.05) although these times are larger in magnitude than those observed for the no-filter case. The model transfer function with the stabilized filter is:

$$\frac{\Theta_2(s)}{\Theta_1(s)} = \frac{1560 \cdot P(E)}{s^3+(25+\alpha)s^2+(25\alpha)s+1560 \cdot p(E)}$$

where α is four and P(E) is the Gaussian density function (discussed in IIA) with an input of 'E'.

C. STABILIZED FILTER RESPONSE FOR LARGE INPUT DEVIATION

The state space program of IIIB was again utilized to analyze the step response for large input deviation (i.e. deviation out of the R.O.E.). The observed responses were comparable to the intuitive example discussed in IIB but the non-linearities of the filter and the extended complexity of second-order dynamics created more complex responses. The filter did cause the system to react in a slow manner to inputs out of the R.O.E. but created two distinctive output families. These families, along with the one for the linear R.O.E., are shown in Table 1. The table represents data taken over the range of input deviation from zero to 3.5. The Pure Probability Effect family is characterized by a relatively constant overshoot of 0.344 to 0.347σ. Data taken for input deviation between 3.5 and 4.0 also exhibited the relatively constant overshoot phenomenon of Family III but the magnitudes were slightly smaller. A computer generated example of each family response is shown in Figures 8, 9, and 10. It can be seen that as the input deviates more and more from the R.O.E. boundary value of σ the rise time increases rapidly. Between the rise time and the settling time of steady state the response is linear

Table 1

Family Number	Type	Input Range (σ)	Rise Time Range (Sec)	Overshoot Range (σ)
I	Linear	$o<\sigma<1$	Constant at 0.8	.172-.28
II	Hybrid	$1\leq\sigma<1.9$	0.85 to 1.2	.286-.343
III	Pure Probability Effect	$1.9\leq\sigma\leq3.5$	1.25 to 9.49	Constant at 0.346

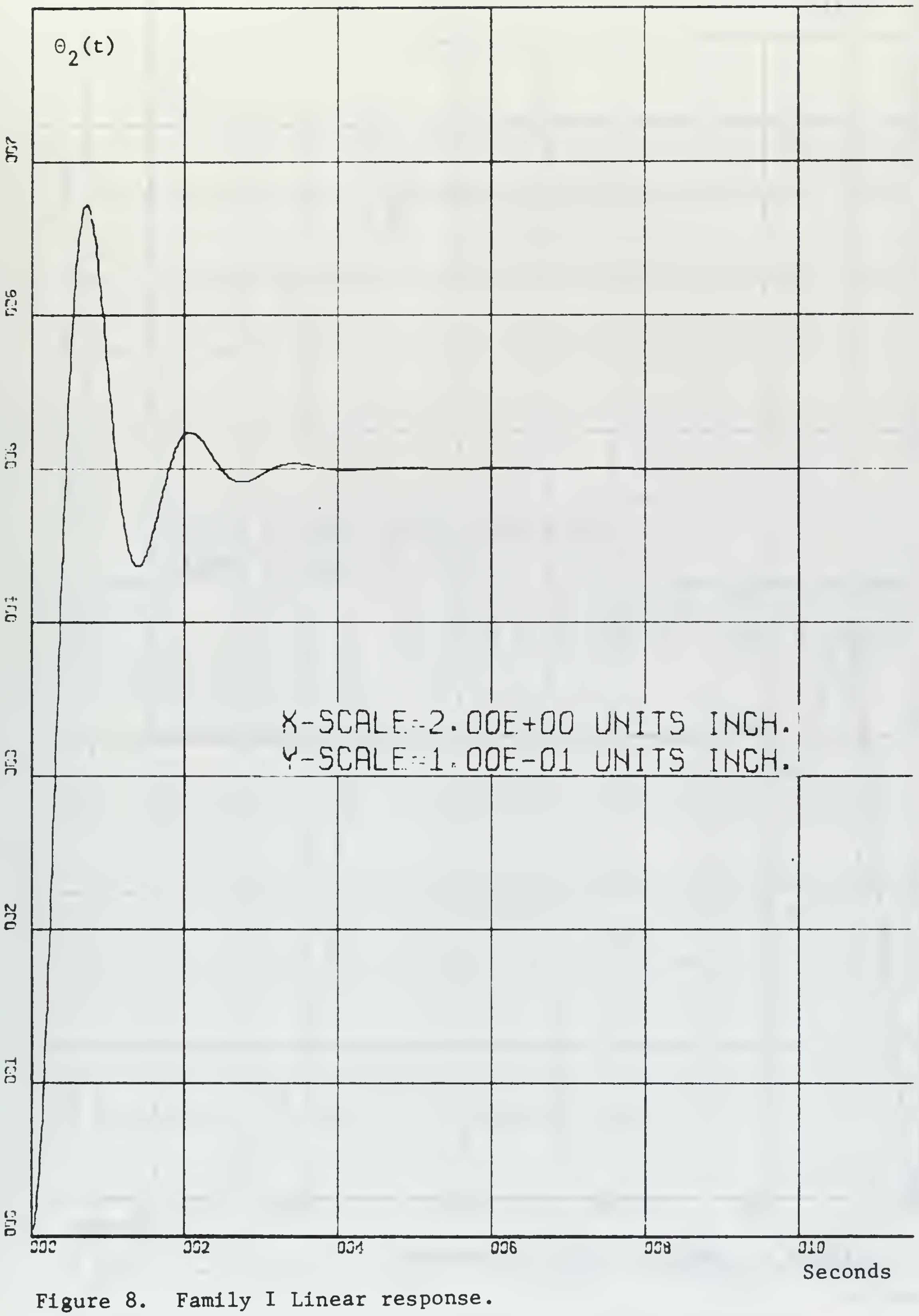


Figure 8. Family I Linear response.

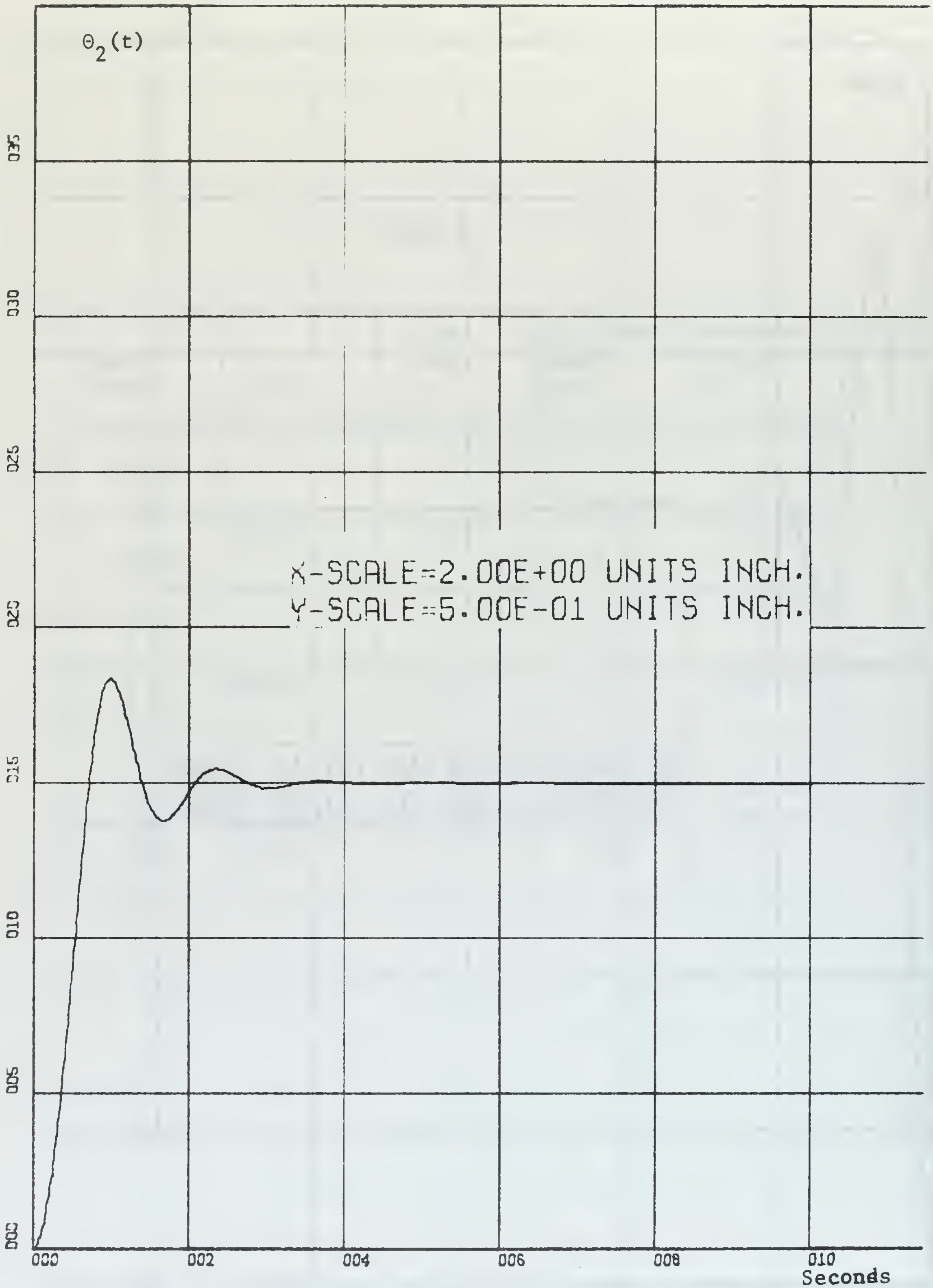


Figure 9. Family II Hybrid response.

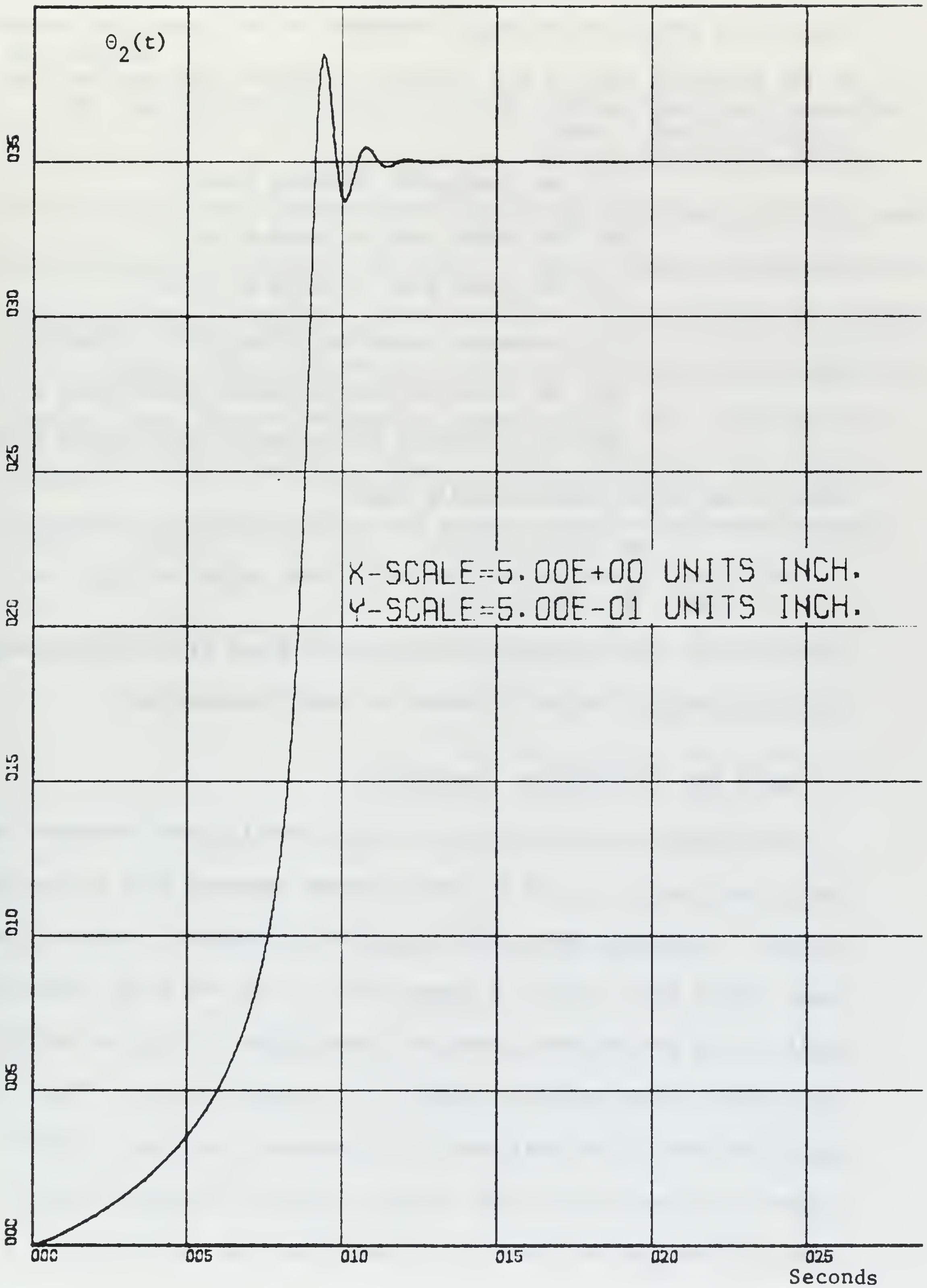


Figure 10. Family III Pure Probability Effect response.

and oscillates with exponential decay toward the final value. Rise time and overshoot characteristics of the system with the filter included were seen to be strongly dependent on the inputs and parameters of the system as well as the choice of feedback magnitude for the integrator block. Given,

$\alpha \equiv$ The integrator feedback factor

$\beta \equiv$ The input step in range $o<\sigma\leq 4.0$.

$\gamma \equiv$ The input step in range $o<\sigma<1.9$.

$\underset{\sim}{\delta} \equiv$ Parameter array describing system dynamics.

$\varphi \equiv$ The Rise Time for responses with filter in system.

$\vartheta \equiv$ The overshoot for responses with filter in system.

Then, it was found experimentally that:

$$\varphi = f(\underset{\sim}{\delta},\ \alpha,\ \beta)$$

$$\text{and} \quad \vartheta = g(\underset{\sim}{\delta},\ \alpha,\ \gamma)$$

The effect of the independent variables on φ and ϑ is complicated and was not studied in depth to obtain an exact formulation.

D. ERROR DUE TO NUMERICAL INTEGRATION

According to the Final Value Theorem from Laplace Transform theory the steady-state error of the model system response with a step input is zero. Utilizing numerical integration techniques, however, produced finite error values at steady state. For the model used in this analysis the errors were essentially negligible for inputs below σ but reached values peaking at $6\text{x}10^{-5}\sigma$ for higher inputs. These errors may be factors to be considered if a computer directing a control system is called upon to make precise, logical decisions based on numerical integration results. It was found that the system steady-state errors with the probability filter added were appreciably less than those errors for the no-filter case in the region $0<\sigma<2.7$.

E. SUMMARY

For a given system, $\underset{\sim}{\delta}$, the α filter feedback term must be manipulated to produce the desired family of responses over the entire β input range. For a given set of engineering requirements, it may also be necessary to optimize $\underset{\sim}{\delta}$ in order to realize desired characteristics for the response families. For the filter to be effective the response in the R.O.E., where the desired signal is situated, must be made very nearly linear and the response to inputs out of this range must be sluggish. This provides in theory a filtering scheme that readily responds to desirable inputs and rejects unwanted information since the rise time before reaching an effective value is so large.

IV. PROBABILITY FILTERING IN JAMMING ENVIRONMENT

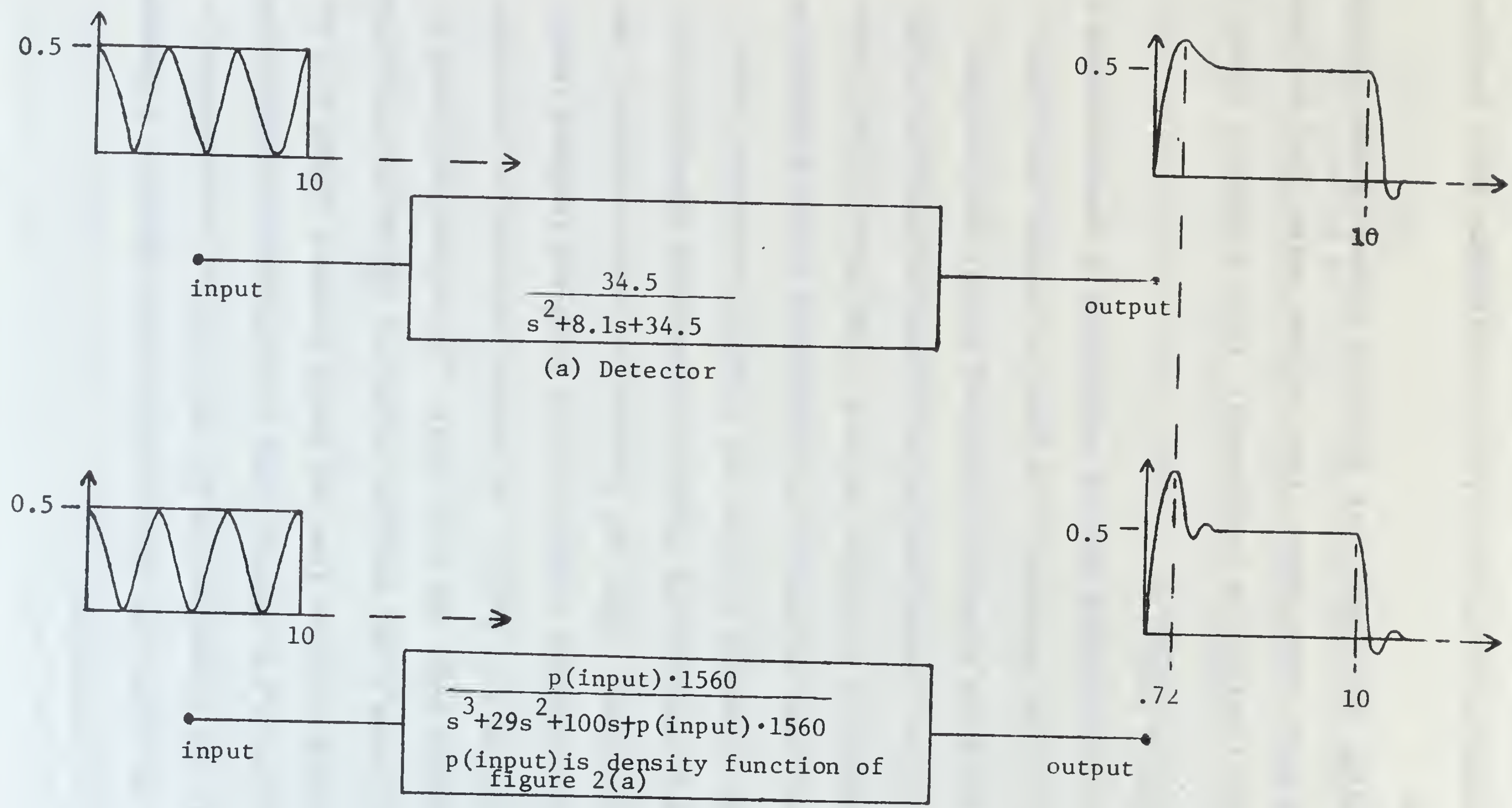
A. ANALYSIS MODEL

The preceding portions of this study have discussed the theory and proper implementation of the probability filter. It now remains to discuss the filter's performance when an "expected" or information signal is subjected to noise jamming. The model used for this experiment consisted of two transfer functions formulated to represent two envelope detectors. One of the detectors had a stabilized probability filter for its front end and one did not. It was assumed that the inputs to these transfer function "detectors" were intermediate-frequency (i-f) radar echoes. A pulse train was established to represent a high data rate of long duration echoes, each having a magnitude of 0.5 volts (well within the R.O.E.). In the model simulation the pulse duration was ten seconds and the pulse repetition time was 15 seconds. This model was computer programmed utilizing the IBM 360 Digital Simulation Language (DSL) with the integration being accomplished by the fifth-order Milne predictor-corrector method. This language was used primarily because it offers a wide range of noise generation options. Included are Gaussian noise with variable deviation, and uniform distribution noise with variable ranges. The ability to simulate these various noise voltages at the front end of the model detectors is a very good approximation to what happens when an actual receiver is subjected to certain jamming conditions. One effective method of producing Gaussian statistics at the output of a receiver filter is to jam with a signal whose carrier is frequency modulated by wide band noise. Similarly, jamming devices attempt to obtain a uniform or "whitened" jamming power spectrum by multiplying the voltages of a Gaussian noise modulated carrier, X, by the well known Error Function, $\frac{2}{\sqrt{\pi}} \int_o^x e^{-\mu^2} \, d\mu$.

In a noiseless environment both detectors produced almost identical outputs to the pulse train input. Thus, without noise, both detectors were practically identical in performance. This is shown in Figure 11.

B. PRESENTATION OF DATA

The first test conducted was the application of Gaussian noise of unit variance to both detectors. The detector without the filter attempted to track the signal but exhibited erratic oscillations, going negative upon occasion during the information pulse duration. The output remained in a corridor between 0.348 and 0.652 volts for only <u>40%</u> of the pulse duration. The probability filter detector, on the other hand, attempted to track the information voltage of 0.5 volts relatively smoothly and oscillated about this value for the duration of the signal. When the information pulse terminated, the detector output "wiggled" toward the zero level and remained there, or below, until the next pulse. The maximum deviation of the output about the 0.5-volt input was 0.152 volts. The output oscillated in the corridor between 0.348 and 0.652 volts for <u>70%</u> of the pulse duration. After 8.52 seconds of pulse time the output suddenly dipped to a low value of 0.079 volts but promptly rose to within the corridor where it remained until the termination of the input pulse. After initial orientation, the filter was able to track, quite smoothly, successive pulses from the information train.



(a) Detector

(b) Detector with probability filter.

Figure 11. Jamming Analysis Model

The filter feedback factor, α, for the above test was four. The same noise jamming was also applied with the feedback factor set at six. Since the detector was designed to have a linear response in the R.O.E. with an α of four the increase in feedback destroyed this condition. However, it was found that the information tracking ability of the filter in the presense of noise was increased. The response stayed in a smaller corridor than before, 0.348-0.63 volts, for 75% of the pulse duration and tracked succeeding pulses with reduced overshoots. This improvement was observed up to an α of eight and appears to be a method of "fine tuning" the filter response in noisy situations providing the sacrifice of linearity and, hence, the proper reproduction of noiseless information is acceptable.

The second test was the injection into the probability detector of heavy Gaussian noise along with the echo pulse train. The density function of the noise had a variance of sixteen. In the presence of this major disturbance the filter was not able to pick out the information pulses but stayed "quiet" and did not respond to positive noise voltages. Thus, the noise was able to suppress the information voltages but could not make the detector overflow with erroneous tracking.

The filter was next subjected to a uniform noise jamming spectrum covering the range -2 to +2 volts. In the presence of this effective form of jamming the filter was not able to "readout" the pulse train very accurately, but did not give many false alarm jumps above the 0.5 volt level. The maximum false alarm level indicated was 1.18 volts and occurred during the first pulse. The detector seemed better able to track the second and succeeding pulses and oscillated in a corridor between 0.2 and 0.8 volts for most of the pulse durations.

Probability filtering was also tested with the same uniform noise jamming discussed above but without the information pulse train. The output of the filtered detector was viewed over an interval of 20 seconds and throughout the observation time the response attempted to seek the zero level. The maximum deviation above the zero level was 0.105 volts and the signal remained in a corridor bounded by zero and 0.04 volts for 87% of the viewing interval. Thus, since no voltages within the R.O.E. were persistent enough to persuade the filter that an "expected" value was present the device "tracked" the zero, no-information level.

V. CONCLUSION

The concept of probability filtering can be an effective and relatively simple way of achieving signal recognition under non-ideal reception conditions. In order to implement such a filtering device inherent stability problems must be overcome. For stability, the filter should be accurately matched to the system it is supporting by manipulation of the integration block feedback. In addition, linear and non-linear response ranges must be achieved through knowledge of expected filter inputs and choice of a proper probability density function. If the filter can be stabilized properly good noise rejection and signal tracking can be expected. In the presence of Gaussian noise with an rms value twice as large as the information signal amplitude, the signal detection with probability filtering was seen to be almost twice as effective as that without filtering. With very heavy jamming applied the filter was not able to track the signal but did not give erroneous information and remained dormant.

Due to its good noise suppression characteristics the filter may also serve a useful function as the front end of correlation filters, reducing the computer matrix calculations of the system.

The non-linear construction blocks of the filter negate strict requirements on linearity in power sources and other components of the system being filtered. Non-linear behavior of devices may also be exploited to act as the filter's weighting function, giving the system designer added flexibility in realizing performance specifications.

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13. ABSTRACT

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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Computer simulation						
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